W. Najm W.N.
J. Kasamani

AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
MECH 230 - DYNAMICS – QUIZ 1 – SPRING 2013

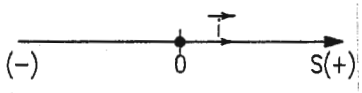
NAME: **ID No:** **Mar. 14, 2013**

1.5 hours closed book quiz

- Solve on the answer booklet in sequence.
- Question sheet will not be corrected and must be returned.
- Write clearly. Clarity is important in grading.
- Vectors are indicated in bold.
- Take g=9.81 m/s^2.

No.1 – (25%)

The rectilinear motion of a particle is defined by $S=5t^2-100t$, where S and t are expressed in meters and seconds.



5% 1- Determine the position, velocity, and acceleration of the particle at t=0 s.

5% 2- Determine the instant of time and the position of the particle when it changes its sense of direction of motion.

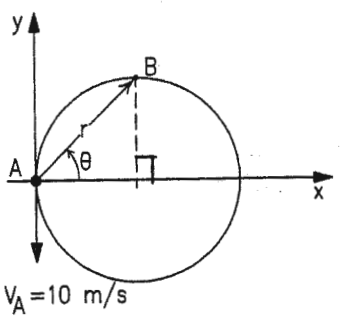
5% 3- Determine the displacement of the particle between t=5 s and t=30 s.

5% 4- Determine the average velocity of the particle between t=5 s and t=30 s.

5% 5- Determine the distance travelled and the average speed of the particle between t=5 s and t=30 s.

No.2 – (25%)

A particle starts its circular motion at A with a speed of 10 m/s as shown. The speed of the particle increases at a constant rate of 2 m/s^2. The particle moves counterclockwise along the circular path of radius 100 m.



10% 1- Determine the distance travelled, the time, and the speed of the particle when it reaches position B the first time.

5% 2- Determine the tangential and normal acceleration of the particle at position B.

5% 3- Determine the radial and transverse components of velocities of the particle at position B. (Show the vectors on a clear diagram).

5% 4- Determine $r, \dot{r}, and\ \theta, \dot{\theta}$ when the particle reaches position B for the first time.

USE FOR ANSWER

PROBLEM 1

Given: rect. motion $S = 5t^2 - 100t$; $s \rightarrow m$, $t \rightarrow s$

1. Find s_0, V_0, a_0 @ $t=0$

Solution:

$$v = \frac{ds}{dt} = 10t - 100 \; ; \quad a = \frac{dv}{dt} = 10$$

@ $t=0$; $s_0 = 0$ m ; $V_0 = -100$ m/s ; $a_0 = 10$ m/s^2 = Constant.

2. change in sense of direction $\Rightarrow V_{10} = 0$

$10t - 100 = 0$; $t = \frac{100}{10} = 10$ s } @ $t = 10$ s

3. Find $\vec{\Delta s}_{5 \rightarrow 30} = \vec{s}_{30} - \vec{s}_5$;

$s_5 = 5(5)^2 - 100(5) = -375$ m

$s_{10} = 5(10)^2 - 100(10) = -500$ m

$s_{30} = 5(30)^2 - 100(30) = 1500$ m

$t = 0$; $s_0 = 0$

$t = 5_s$; $s_5 = -375$ m

$t = 10_s$; $s_{10} = -500$ m

$t = 30$; $s_{30} = 1500$ m

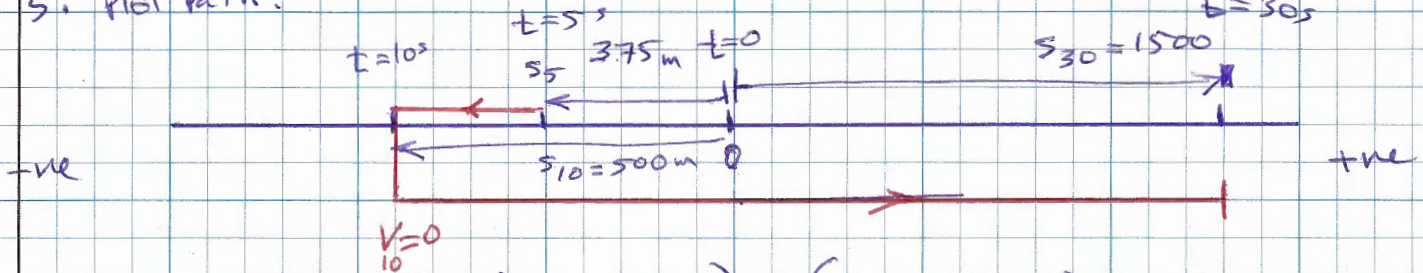
$\Delta s_{5 \rightarrow 30} = s_{30} - s_5 = 1500 - (-375) = 1875$ m $\rightarrow$ Ans.

$\vec{\Delta s}_{5 \rightarrow 30} = 1875$ m $\rightarrow$ Ans.

4. $\vec{V}_{avg.} = \frac{\vec{\Delta s}_{5-30}}{\Delta t} = \frac{s_{30} - s_5}{30 - 5} = \frac{1875 \text{ m}}{25_s} = 75$ m/s Ans.

$\vec{V}_{avg.} = 75$ m/s $\rightarrow$ Ans.

5. Plot Path:



distance traveled $= (500 - 375) + (500 + 1500)$

$= 125 + 2000 = 2125$ m ; $S_{T_{5-30}} = 2125$ m Ans

$V_{sp.avg} = \frac{S_T}{\Delta t} = \frac{2125}{30-5} = \frac{2125 \text{ m}}{25_s} = 85$ m/s

$V_{sp.avg} = 85$ m/s Ans.

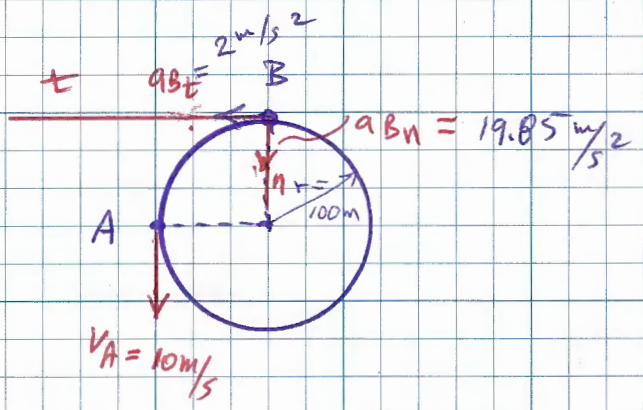
USE FOR ANSWER

PROBLEM 2

Given: $V_A = 10 m/s \downarrow$

$a_t = 2 m/s^2$

$r = 100 m$ (Circle radius)



1. Find S_{AB} ; t_{AB} , V_B

$$S_B = S_{0_A} + V_{0_A} t_{AB} + \frac{1}{2} a_t t_{AB}^2$$

$$S_B = 0^{\text{origin}} + 10 t_{AB} + \frac{2}{2} t_{AB}^2 \; ; \; S_B = 10 t_{AB} + t_{AB}^2$$

$$S_B = r\theta = 100 \left(3\pi/2\right) = 150\pi = 471.239 m$$

$S_B = 471.239 m$ Ans.

$$t_{AB}^2 + 10 t_{AB} - 150\pi = 0 \; ; \; t_{AB} = 17.2764 s \quad \text{Ans.}$$

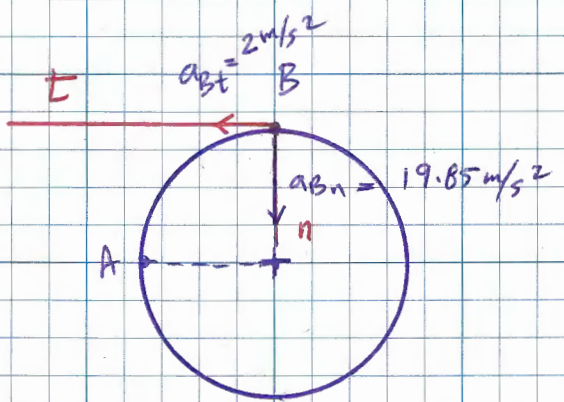
$$V_B = V_A + a_t t_{AB} \; ; \; V_B = 10 + 2(17.2764) = 44.5528 m/s$$

$V_B = 44.5528 m/s$ Ans.

2. Find a_{Bt} & a_{Bn}

$a_{Bt} = 2 m/s^2$ given (Constant) Ans.

$$a_{Bn} = \frac{V_B^2}{\rho_B} = \frac{(44.5528)^2}{100} = 19.8495 m/s^2 \quad \text{Ans.}$$



3. Find a_r, a_θ @ B

Project a_t & a_n along r.

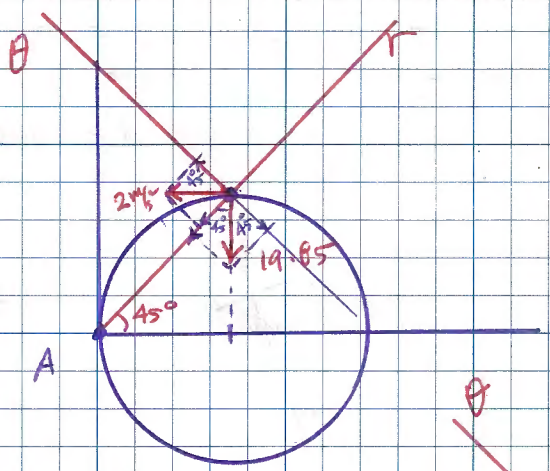
$a_r =$

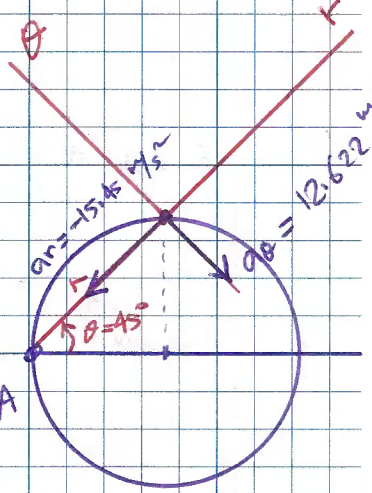
$-19.85 \angle 45^\circ - 2 \angle 45^\circ$

$a_r = -15.4503 \text{ m/s}^2$ Ans.

$a_\theta = 2 \angle 45^\circ - 19.85 \angle 45^\circ$

$a_\theta = -12.622 \text{ m/s}^2$ Ans.





Plot separately :

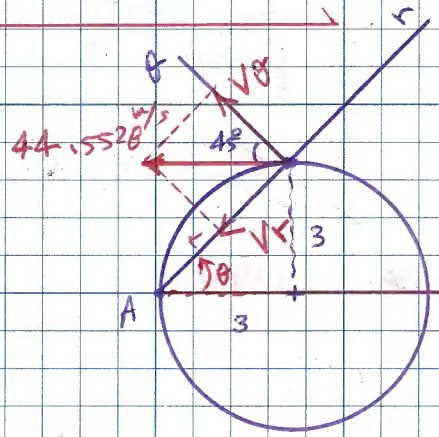
4. Find V_r, V_θ @ B

$V_r = -44.5528 \angle 45^\circ$

$V_r = -31.504 \text{ m/s}$

$V_\theta = 44.5528 \angle 45^\circ$

$V_\theta = 31.504 \text{ m/s}$



5. Find r, $\dot{r}$ $\ddot{r}$

$r = \sqrt{2}(100) = 4.24264 \text{ m}$ Ans.

$\dot{r} = V_r = -31.504 \text{ m/s}$ Ans.

$a_r = \ddot{r} - r\dot{\theta}^2$; $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$-15.4503 = \ddot{r} - 4.24264 (7.4256)^2$

$\ddot{r} = 218.4869 \text{ m/s}^2$ Ans.

θ, $\dot{\theta}$, $\ddot{\theta}$

$\theta = 45^\circ = \pi/4$ rad.

$V_\theta = r\dot{\theta}$; $\dot{\theta} = \frac{V_\theta}{r}$

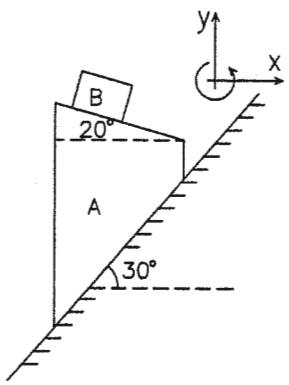
$\dot{\theta} = \frac{31.504}{3\sqrt{2}} = \frac{31.504}{\sqrt{2}(100)} = 7.4256 \text{ rad/s}$

$a_\theta = -12.622 = 4.24264\,\ddot{\theta} + 2(-31.504)(7.4256)$

$\ddot{\theta} = 107.3035 \text{ rad/s}^2$ Ans.

No.3 – (25%)

Block B of mass 20 kg rests as shown on the upper surface of the 30 kg wedge A. The system is released from rest and friction is neglected between all surfaces of contact.



5% 1- Draw the free body diagrams and the kinetic diagrams of the block and the wedge separately.

5% 2- Write and project along the x-y axes the kinematic equation of accelerations between block B and wedge A.

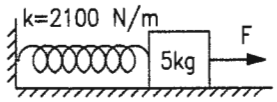
5% 3- Write and project along the x-y axes the kinetic equation of motion **F=ma** of block B.

5% 4- Write and project along the x-y axes the kinetic equation of motion **F=ma** of wedge A.

5% 5- Determine the acceleration of block B and wedge A. Show clearly in a matrix form the coefficient of the unknowns of the n-equation, n-unknown.

No.4 – (25%)

The 5 kg block is attached to an unstretched spring of constant k=2100 N/m. The coefficient of static and kinetic friction between the block and the plane is 0.40. If a force F is slowly applied to the block until the tension in the spring reaches 100 N and then suddenly removed.



10% 1- Draw the free body diagram of the block just after the 100 N force is removed.

5% 2- Determine the speed of the block as it returns to its initial position.(unstretched position of the spring).

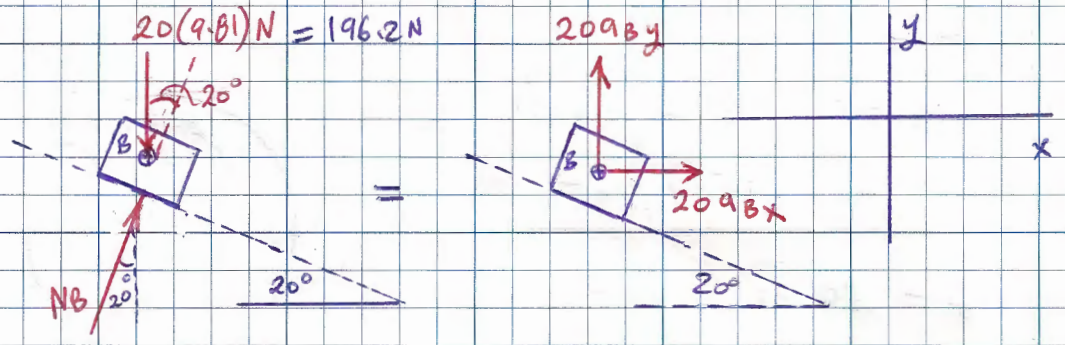
10% 3- Determine the maximum speed of the block.

USE FOR ANSWER

PROBLEM 3

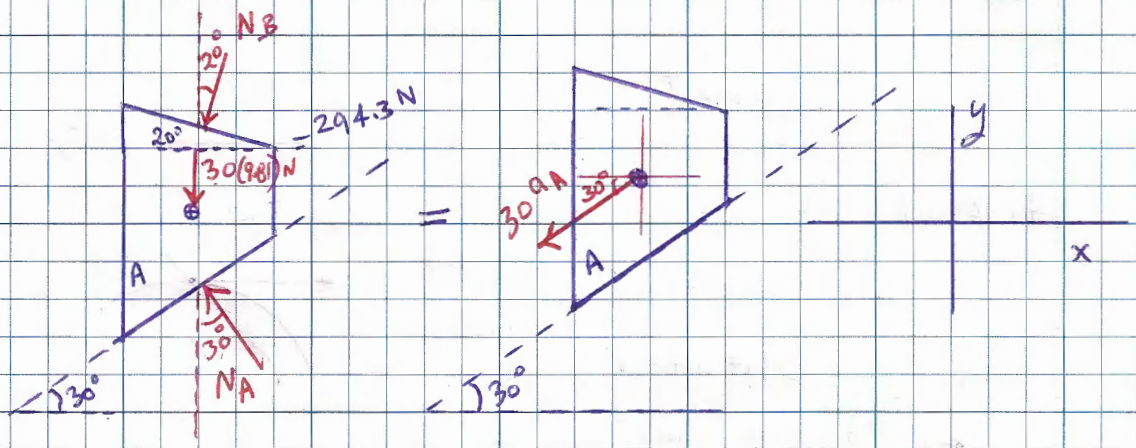
Given $m_A = 30\,kg$ (wedge), $m_B = 20\,kg$ (Block); initially @ rest & no friction

1- F.B.D: B



F.B.D: A

Unknowns:
a_{Bx}, a_{By}, a_A
$N_B, N_A, a_{B/A}$



2- Relative velocity eqn.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\stackrel{+}{\rightarrow} a_{Bx} = -a_A \cos 30^\circ + a_{B/A} \cos 20^\circ \quad \checkmark$$

$$+\uparrow a_{By} = -a_A \sin 30^\circ - a_{B/A} \sin 20^\circ$$

3- on Block B:

$$\stackrel{+}{\rightarrow} \quad N_B \sin 20^\circ = 20\, a_{Bx} \quad \checkmark$$

$$+\uparrow \quad N_B \cos 20^\circ - 196.2^N = 20\, a_{By} \quad \checkmark$$

4- on Block A:

$$\stackrel{+}{\rightarrow} \quad -N_A \sin 30^\circ - N_B \sin 20^\circ = -30\, a_A \cos 30^\circ = -25.980\, a_A$$

$$+\uparrow \quad N_A \cos 30^\circ - 294.3 - N_B \cos 20^\circ = -30\, a_A \sin 30^\circ$$

$$= -15\, a_A$$

$$-0.5N_A - 0.34202N_B + 25.9808a_A + 0 + 0 + 0 = 0$$

$$0.86603N_A - 0.9397N_B + 15a_A + 0 + 0 + 0 = 294.3$$

$$0 + 0.34202N_B + 0 - 20a_{Bx} + 0 + 0 = 0$$

$$0 + 0.9397N_B + 0 + 0 - 20a_{By} + 0 = 196.2$$

$$0 + 0 + 0.86603a_A + a_{Bx} + 0 - 0.9397a_{B/A} = 0$$

$$0 + 0 + 0.5a_A + 0 + a_{By} + 0.34202a_{B/A} = 0$$

$$\begin{bmatrix} -0.5 & -0.34202 & 25.9808 & 0 & 0 & 0 \\ 0.86603 & -0.9397 & 15 & 0 & 0 & 0 \\ 0 & 0.34202 & 0 & -20 & 0 & 0 \\ 0 & 0.9397 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0.86603 & 1 & 0 & -0.9397 \\ 0 & 0 & 0.5 & 0 & 1 & 0.34202 \end{bmatrix} \begin{bmatrix} N_A \\ N_B \\ a_A \\ a_{Bx} \\ a_{By} \\ a_{B/A} \end{bmatrix} = \begin{bmatrix} 0 \\ 294.3 \\ 0 \\ 196.2 \\ 0 \\ 0 \end{bmatrix}$$

$N_A = 305.333\ N$

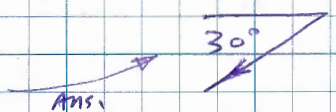
$N_B = 78.506\ N$

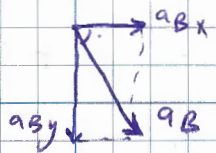
$a_A = 6.9096\ m/s^2$

$a_{Bx} = 1.3425\ m/s^2$

$a_{By} = -6.1214\ m/s^2$

$a_{B/A} = 7.7966\ m/s^2$





$\vec{a}_B = 1.3425\hat{i} - 6.1214\hat{j}\ m/s^2$ Ans.

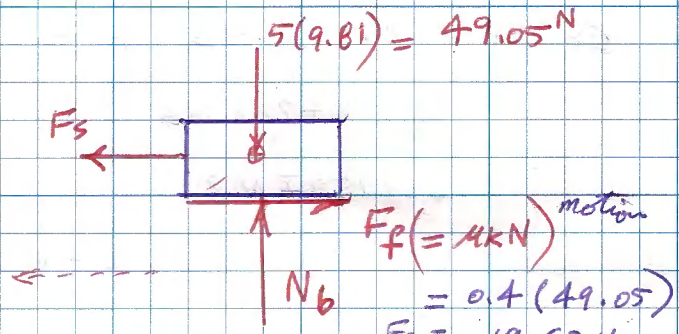
USE FOR ANSWER

PROBLEM 4:

Given $m_b = 5\,kg$; $K = 2100\ N/m$; $\mu_s = 0.6$; $\mu_k = 0.4$

initially $F_s = 100N$.

1. F.B.D : Block



$F_s = KS_1$

$100N = 2100\ N/m\ S_1$

$S_1 = 0.04762\ m$

$\rightarrow \Sigma F_x = 0$; $-100 + F_f = 0$; $F_f = 100N$; $F_{f_{max}} = 0.6(49.05) = 29.43\ N$

$\uparrow \Sigma F_y = 0$; $N_b = 49.05\ N$; $F_f = 100N > F_{max} = 29.43N \Rightarrow$ motion exists.

$\therefore F_f = \mu_k N_b = 0.4(49.05)$

$F_f = 19.62\ N$

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2. (vel., displ., Force) $\Rightarrow$ use $T_1 + \Sigma U_{1-2} = T_2$

$T_1 = 0$ rest ; $T_2 = \frac{1}{2} m V_2^2 = \frac{1}{2}(5) V_2^2 = 2.5 V_2^2$

$\Sigma U_{1-2} = -19.62(0.04762\,m) + \frac{1}{2} k s_1^2 - \frac{1}{2} k s_2^2$ (s_2 → unstretched)

$= -0.9343\ J + \frac{1}{2}(2100)(0.04762)^2$

$= -0.9343 + 2.381 = 1.44675\ J$

$\therefore 0 + 1.44675 = 2.5 V_2^2$; $V_2 = 0.7607\ m/s$ Ans.

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3. Find V_{max}.

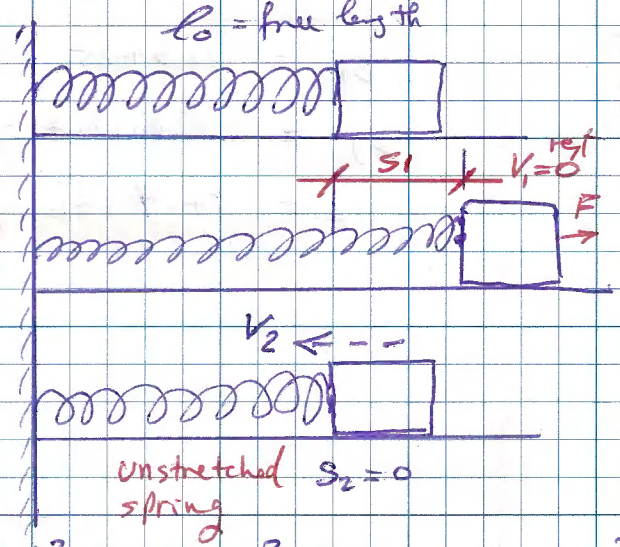
@ $V_{max} \Rightarrow \frac{dV}{dt} = a = 0$

$\leftarrow \Sigma F_x = ma_x$

$F_s - 19.62 = 0$; $F_s = 19.62\ N$

$KS_2 = 19.62$

$2100\ S_2 = 19.62$; $S_2 = 9.3429 \times 10^{-3}\ m$



$\therefore$ rest @ V_{max} : $0 - 19.62\left(0.04762 - 9.3429 \times 10^{-3}\right) + \frac{1}{2}(2100)(0.04762)^2 - \frac{1}{2}(2100)(9.3429 \times 10^{-3})^2 = 2.5 V_{max}^2$; $2.1061 = 2.5 V_{max}^2$

$V_{max} = 1.1028\ m/s$ Ans.